# AMBIGUITY OF THE HYDRODYNAMIC CHARACTERISTICS (HDC) OF BOILING CHANNELS IN CRYOGENIC UNITS

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Criteria are obtained for the ambiguity of HDC of boiling channels which allow for local inlet and outlet drag. Results of calculations presented for the most commonly used cryogens indicate the possible ambiguity of the HDC.

Cryogenic systems in which a two-phase flow of a cryogenic agent is forced to circulate in the channels of the unit, removing heat, have recently come into use [1, 2]. Various types of instability, characterized by variation of the flow rate, pressure, and other parameters, are possible in such systems. When there is a change in the aggregate state in the channels, one of the reasons for the instability is the ambiguity of the hydrodynamic characteristics (HDC) of the channels.* An analytic expression was obtained in [3, 4] for the HDC of a boiling channel in the form of a third-degree polynomial in the flow rate. This polynomial was used to show that the HDC may be ambiguous under certain conditions. This is shown in Fig. 1, where the quantity $\Delta P_1$ satisfies three values of flow rate $G_1$, $G_2$, and $G_3$. The authors of [3, 4] obtained the following nonambiguity (stability) condition for a straight channel without local resistances and with a uniform heat supply along the channel and constant parameters at the channel inlet:

$$\Delta i_{\text{in}} \leqslant 7.46 \frac{r}{\left(\frac{\rho'}{\rho''} - 1\right)} \quad \text{or} \quad \text{Ja} \leqslant 7.46. \tag{1}$$

In actuality, there are always local resistances along the channels. We therefore analyzed the HDC of a boiling channel with local resistances at the inlet and outlet (Fig. 2). The flow in such a channel enters in the form of a liquid heated to a temperature below the saturation temperature and becomes two-phase under the influence of a thermal load Q. The section where the flow reaches the saturation state is located the following distance from the inlet:

$$L_s = \frac{G\Delta i_{\text{in}}}{Q} L. \tag{2}$$

The drag of the channel is written thus:

$$\Delta P = \Delta P_{\text{in}} + \Delta P_{\text{sp}} + \Delta P_{\text{tp}} + \Delta P_{\text{out}}. \tag{3}$$

The following assumptions were made to determine the conditions of nonambiguity of the HDC: the flow is uniform, equilibrium, and homogeneous; the properties of the flow (medium) are independent of temperature and pressure; flow acceleration effects are negligible and are ignored; the drag coefficient is constant and is the same for single- and two-phase flows; the flow at the outlet is always two-phase, $0 \leqslant x_{\text{out}} \leqslant 1$; the heat flow is uniform along the channel; the input parameters of the flow are constant and are independent of flow rate. Allowing for the foregoing, Eq. (3) takes the form

$$\Delta P = \xi_{\text{in}} \frac{G^2}{2\rho' F^2} + \lambda \frac{L_s}{d} \frac{G^2}{2\rho' F^2} + \lambda \frac{(L - L_s)}{d} \frac{G^2}{2\rho' F^2} \left[1 + \frac{x_{\text{out}}}{2}\left(\frac{\rho'}{\rho''} - 1\right)\right] + \xi_{\text{out}} \frac{G^2}{2\rho' F^2}\left[1 + x_{\text{out}}\left(\frac{\rho'}{\rho''} - 1\right)\right]. \tag{4}$$

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*By the HDC of a boiling channel, we mean the dependence of the drag $\Delta P$ on the flow rate G of the working fluid.

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TABLE 1. Characteristic Values of Jacob Numbers for Different Substances in Relation to Inlet Temperature and Pressure

| $P_{in}$, bars | $T_{in}$, K | Ja | $P_{in}$, bars | $T_{in}$, K | Ja |
|---|---|---|---|---|---|
| Helium | | | Hydrogen | | |
| 0,2 | 2,5 | 1,28 | 1,0 | 14,0 | 5,96 |
| 0,4 | 2,5 | 1,69 | 1,0 | 16,0 | 4,21 |
| 0,5 | 2,5 | 1,71 | 2,0 | 16,0 | 3,85 |
| 0,6 | 2,5 | 1,70 | 4,0 | 16,0 | 3,15 |
| 0,8 | 2,5 | 1,66 | 6,0 | 16,0 | 2,72 |
| 1,0 | 2,5 | 1,61 | 8,0 | 16,0 | 2,42 |
| 1,4 | 2,5 | 1,51 | 10,0 | 16,0 | 2,15 |
| 1,8 | 2,5 | 1,40 | 12,0 | 16,0 | 1,88 |
| 2,2 | 2,5 | 1,26 | | | |
| | | | Water | | |
| Nitrogen | | | 1,0 | 273,15 | 300,02 |
| 1,0 | 70,0 | 12,84 | 20,0 | 313,15 | 32,72 |
| 6,0 | 70,0 | 9,34 | 40,0 | 313,15 | 20,73 |
| 14,0 | 70,0 | 5,81 | 90,0 | 313,15 | 11,59 |
| 20,0 | 70,0 | 4,64 | 130,0 | 313,15 | 8,57 |
| 30,0 | 70,0 | 3,38 | 180,0 | 313,15 | 6,15 |

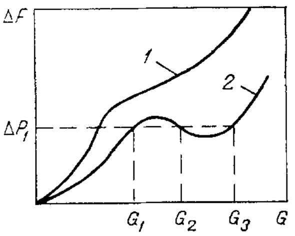

Fig. 1

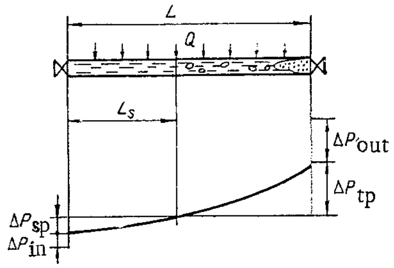

Fig. 2

Fig. 1. Examples of possible hydrodynamic characteristics of a boiling channel: 1) unambiguous; 2) ambiguous.

Fig. 2. Diagram of boiling channel and pressure-gradient distribution within.

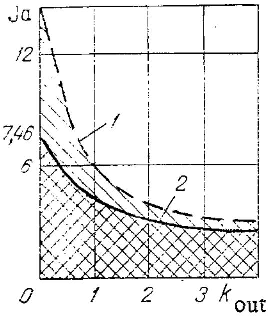

Fig. 3. Region of unambiguous and ambiguous hydrodynamic characteristics of a boiling channel with different values of $k_{in}$: 1) $k_{in} = 1$; 2) 0. The value of Ja = 7.46 corresponds to the Ledinegg–Petrov criterion.

Substituting Eq. (2) into (4) and introducing scales of the physical quantities, we obtain the HDC of the channel in the form of a third-degree polynomial in the flow rate:

$$\Delta\pi = \frac{1}{2}\mathrm{Ja}^2 m^3 + [(1 + k_{out})(1 - \mathrm{Ja}) + k_{in}]\, m^2 + \left(\frac{1}{2} + k_{out}\right) m. \tag{5}$$

It follows from Fig. 1 that, when the HDC of the channel is ambiguous, the derivative $\partial(\Delta\pi)/\partial m$ should also have a negative value, i.e.,

$$\frac{\partial(\Delta\pi)}{\partial m} = \frac{3}{2}\mathrm{Ja}^2 m^2 + 2[(1 + k_{out})(1 - \mathrm{Ja}) + k_{in}]\, m + \left(\frac{1}{2} + k_{out}\right) \leqslant 0. \tag{6}$$

The earlier assumption $0 \leqslant X_{out} \leqslant 1$ can be written as

$$\frac{1}{\mathrm{Ja} + \left(\frac{\rho'}{\rho''} - 1\right)} \leqslant m \leqslant \frac{1}{\mathrm{Ja}}. \tag{7}$$

Solving the inequalities (6) and (7), we obtain the following conditions required for the existence of ambiguity of the HDC of a boiling channel:

$$\mathrm{Ja} \geqslant \left(1 + \frac{k_{in}}{1 + k_{out}}\right) \frac{1}{1 - \sqrt{\frac{3}{4}\frac{(1 + 2k_{out})}{(1 + k_{out})^2}}} \quad \text{at } k_{out} < 1, \tag{8}$$

$$\mathrm{Ja} \geqslant 2\frac{(1 + k_{out} + k_{in})}{k_{out}} \quad \text{at } k_{out} > 1. \tag{9}$$

It follows from (8) and (9) that local resistances at the inlet ($k_{in} > 0$) diminish the region of existence of ambiguity and resistances at the outlet ($k_{out} > 0$) expand this region.

Conditions (8) and (9) are graphed in Fig. 3. The region below the curves for $k_{in} = 1$ or $k_{out} = 0$ is characterized by single-valued HDC's, while the region above is characterized by ambiguous HDC's. In the case of absence of local resistances ($k_{in} = k_{out} = 0$), condition (8) becomes the Ledinegg—Petrov stability criterion (1). The point of intersection of the curve for $k_{in} = 0$ with the y axis in Fig. 3 corresponds to this criterion.

When $k_{out} \to \infty$, the boundary of instability asymptotically approaches Ja = 2. This means that the HDC of the channel can be ambiguous only when $\mathrm{Ja} \geqslant 2$. Proceeding on this basis, let us determine the possibility of ambiguity of the HDC of boiling channels for the working fluids shown in Table 1.

We took an inlet temperature of 2.5°K for helium, which corresponds to the minimum handbook value [5]. An increase in $T_{in}$ lowers Ja, while a decrease in $T_{in}$ below 2.18°K is unacceptable for the model in question because helium becomes superfluid in this region. In connection with this, Table 1 shows values of Ja close to the maximum.* It is apparent from the table that the Ja numbers sometimes reach 2. This means that the HDC of boiling channels is single-valued even when $T_{in}$ is lowered to the temperature at which He-I becomes He-II.

The HDC of boiling channels can be ambiguous when hydrogen or nitrogen are used. This conclusion is supported by experimental data in [6], where the investigators noted abrupt changes in the rate of nitrogen flow through a heated channel at $\mathrm{Ja} \simeq 6$ and $k_{out} \simeq 1000$. At the same time, the HDC of channels with hydrogen may be single-valued at pressures close to critical. Thus, $\mathrm{Ja} \simeq 1.88$ at $P_{in} = 12.0 \cdot 10^5$ Pa and $T_{in} = 16.0$°K.

The data for water presented above for comparison show that its behavior is characterized by relatively high values of Ja. The instability of the HDC of water boiling channels is well known and is taken into account in boiler design practice.

Thus, in designing certain cryogenic units, it is necessary to allow for possible ambiguity of the hydrodynamic characteristics of the boiling channels. The conditions of this ambiguity can be evaluated using the above relations.

## NOTATION

d, equivalent diameter of channel; $i_{in}$ and i', enthalpy of medium at channel inlet and in saturation state; $\Delta i_{in} = i' - i_{in}$; $P_{in}$, pressure of medium at channel inlet; r, specific heat of vaporization; $X_{out}$, mass vapor content of flow at channel outlet; F, cross-sectional area of channel; G, mass flow rate of medium; $G_0$, flow-rate scale, $G_0 = \frac{Q}{r}\left(\frac{\rho'}{\rho''} - 1\right)$; Q, heat supplied to channel; ρ', density of saturated liquid; ρ", density of saturated vapor; $k_{out}$, corrected drag coefficient of channel outlet, $k_{out} = \frac{\xi_{out}}{\lambda} \times \frac{d}{L}$; $k_{in}$, corrected drag coeffiefficient of channel inlet, $k_{in} = \frac{\xi_{in}}{\lambda}\frac{d}{L}$; L, channel length; λ, drag coefficient of chan-

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*When $T_{in}$ is below 2.5°K, the heat capacity of helium is relatively low (≈2 kJ/kgK).

nel; m, dimensionless flow rate, $m = G/G_0$; Ja, modified Jacob number, $Ja = \frac{\Delta i_{in}}{r}\left(\frac{\rho'}{\rho''} - 1\right)$; $T_{in}$, temperature of medium at channel inlet; $\Delta P$, total drag of channel; $\Delta P_0$, pressure scale, $\Delta P_0 = \lambda \frac{L}{d} \frac{G_0^2}{2\rho' F^2}$; $\Delta P_{in}$, drag of channel inlet; $\Delta P_{out}$, drag of channel outlet; $\Delta P_{tp}$, drag of two-phase section of channel; $\Delta P_{sp}$, drag of single-phase section of channel; $\Delta\pi$, dimensionless drag, $\Delta\pi = \Delta P/\Delta P_0$; $\xi_{in}$, $\xi_{out}$, drag coefficients of inlet and outlet.

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